

1 Attorney Docket No. 79866

2
3 NON-LINEAR AXISYMMETRIC POTENTIAL FLOW BOUNDARY MODEL FOR
4 PARTIALLY CAVITATING HIGH SPEED BODIES
5

6 STATEMENT OF GOVERNMENT INTEREST

7 The invention described herein may be manufactured and used
8 by or for the Government of the United States of America for
9 governmental purposes without the payment of any royalties
10 thereon or therefore.

11
12 CROSS-REFERENCE TO RELATED PATENT APPLICATIONS

13 Not applicable.
14

15 BACKGROUND OF THE INVENTION

16 (1) Field of the Invention

17 The present invention relates to computer model of
18 hydrodynamic flows and more particularly, relates to modeling
19 partially cavitating flows over a supercavitating axisymmetric
20 body.

21 (2) Description of the Prior Art

22 Modeling of boundary flows about objects subject to laminar
23 and turbulent flows is well known in the art. High speed
24 underwater vehicles, however, cause cavitation of the surrounding
25 fluid. Cavitation reduces pressure in the fluid below its vapor

1 pressure causing the fluid to vaporize, allowing the undersea
2 vehicle to travel with lower friction when the vehicle is
3 completely surrounded by the cavity.

4 Partial cavitation is an unsteady phenomenon that occurs
5 when part of the supercavitating vehicle is traveling in the
6 cavity. Specifically, this phenomenon occurs during launch of
7 the vehicle. A steady, partial cavitation allows development of
8 vehicle designs which take advantage of drag reduction through
9 cavitation. It may also be possible to take advantage of drag
10 reduction with partial cavitation by properly directing the re-
11 entrant jet that forms in the cavity closure region. Partial
12 cavitation often occurs during maneuvering of the supercavitating
13 vehicle.

14 A slender body theory has been developed to solve
15 axisymmetric supercavitating flows. Using the slender body
16 method, sources are defined along the body-cavity axis and
17 control points along the body-cavity surface. A nonlinear
18 differential equation is formed by imposing dynamic boundary
19 conditions on the cavity. A conical cavity closure is assumed in
20 order to solve the developed nonlinear differential equation.

21 A non-linear boundary element method for determining a
22 cavity shape has been developed. Source and dipole strengths
23 along the body-cavity surface are determined using kinematic
24 boundary conditions on the wetted body surface and dynamic
25 boundary conditions on the assumed cavity shape. The kinematic

boundary condition is then used to update the cavity shape. The process is then iterated to solve for the unknown cavity shape.

Two numerical hydrodynamics models have been developed by the Naval Undersea Warfare Center for axisymmetric supercavitating high speed bodies. These models are the slender body theory (SBT) model and the boundary element (BE) model. Both of these models have been proven to predict cavity shape and parameters with good accuracy.

These models, however, do not account for the transition case when the vehicle is subjected to only partial cavitation.

In the SBT model, total drag is predicted by adding the pressure drag obtained from the model solution and the viscous drag obtained by applying the Thwaites and Falkner-Skan approximations along the wetted portions of the cavitator. This method is extended to subsonic compressible flows using the compressible Green's function. In the BE model, sources and dipoles are defined on the body-cavity shape and are solved using Green's formula. This yields a Fredholm integral equation of the second kind which gives the supercavitating cavity shape.

Partial cavitation modeling has been done by Uhlman, J.S. (1987), The Surface Singularity Method Applied to Partially Cavitating Hydrofoils, Journal of Ship Research, Vol. 31, No. 2, pp. 107-24; Uhlman, J.S. (1989), The Surface Singularity or Boundary Integral Method Applied to Supercavitating Hydrofoils, Journal of Ship Research, Vol. 33, No. 1, pp. 16-20; Kinnas,

1 S.A., and Fine, N.E. (1990), Non-Linear Analysis of the Flow
2 Around Partially and Super-Cavitating Hydrofoils by a Potential
3 Based Panel Method, Proceedings of the IABEM-90 Symposium,
4 International Association for Boundary Element Methods, Rome,
5 Italy, and Kinnas, S.A., and Fine, N.E. (1993), A Numerical
6 Nonlinear Analysis of the Flow Around Two- and Three-Dimensional
7 Partially Cavitating Hydrofoils, Journal of Fluid Mechanics, Vol.
8 254. However, these methods are explicitly adapted for
9 hydrofoils, and the theories presented therein are not readily
10 adapted to supercavitating vehicles.

11 SUMMARY OF THE INVENTION

12 One object of the present invention is a method for modeling
13 partial cavitation.

14 Another object is that such method model partial cavitation
15 about an axisymmetric vehicle.

16 Accordingly, the present invention provides a method for
17 calculating cavity shape for partial cavities about an
18 axisymmetric body having a cavitator located at the foremost end.
19 The method includes receiving system parameter data including
20 geometric data describing the axisymmetric body, a cavity length,
21 and a convergence tolerance. Boundary element panels are
22 distributed along the body-cavity surface and matrices are
23 initialized for each boundary element panel using the unit
24 dipole, unit source functions and known boundary values.
25

1 Disturbance potential matrices are formulated for each boundary
2 element panel using disturbance potentials, normal derivatives of
3 disturbance potentials, and no net flux boundary conditions. The
4 initialized matrices and the formulated matrices are solved for
5 each boundary panel to obtain unknown disturbance potentials
6 along the wetted body-cavity surfaces, and normal derivatives of
7 disturbance potentials along the cavity surface. The cavity
8 position is then updated by moving each panel to satisfy the
9 kinematic boundary condition, no flux across the cavity. The
10 method then tests for convergence against a tolerance, and steps
11 are iterated until convergence is achieved. The method then
12 provides parameters of interest and the location of the cavity as
13 output. Another aspect of this invention allows the calculation
14 of cavity shape and cavity length for an input cavitation number.
15 This is accomplished by an outer loop adjusting cavity length
16 until the model converges to the input cavitation number.

18 BRIEF DESCRIPTION OF THE DRAWINGS

19 These and other features and advantages of the present
20 invention will be better understood in view of the following
21 description of the invention taken together with the drawings
22 wherein:

23 FIG. 1 is a diagram of a partially cavitating axisymmetric
24 body related to the method of the current invention; and

FIG. 2 is a flow chart of the method of the current invention; and

FIG. 3 is a flow chart of another method of the current invention.

DESCRIPTION OF THE PREFERRED EMBODIMENT

FIG. 1 shows a diagram of the physical problem of partial cavitation. FIG. 1 shows a radial cross section of an axisymmetric body 10. Axis r represents the radius from the axis of body 10. Axis x represents the length along the body 10 measured from a cavitator disk 12. Although a cavitator disk is shown, the model can calculate cavities for cavitator cones as well as cavitator disks. Flow, U_∞ , is in the direction of arrow 14. A cavity 16 is shown extending from the edge of the cavitator along the length of body 10. The length of the cavity, ℓ_c , is shown by dimension arrows. Likewise, the length of the body, ℓ_b , is also shown by dimension arrows.

Body 10 extends beyond a cavity closure 18. Cavity 16 is closed to the body 10 with a modified Riabouchinsky cavity termination wall. Cavity closure 18 can be positioned in either body conical section 22 or body cylindrical section 24. The plane of cavity closure 18 is referenced in the following disclosure as an endplate.

Body 10 has a flat front area 20 followed by a conical section 22 and a cylindrical section 24. The diameter of flat

1 front area 20 should be less than or equal to the diameter of the
2 cavitator disk 12 base.

3 The mathematical formulations in of this algorithm are based
4 on using the cavitator diameter to remove dimensionality for all
5 lengths and using the free stream velocity, U_∞ , to remove
6 dimensionality for all velocities. Alternate formulations using
7 standard units can also be developed.

8 The flow field is governed by Laplace's equation,

9
$$\nabla^2 \Phi = 0 \quad (1)$$

10 where Φ is the total potential which is the sum of free
11 stream potential, ϕ_∞ , and disturbance potential, ϕ , giving:

12
$$\Phi = \phi_\infty + \phi \quad (2)$$

13 The free stream potential is the product of the velocity and
14 the distance, x . Because the equation has been non-
15 dimensionalized, the velocity is 1, and the free stream
16 potential, ϕ_∞ , is x . The disturbance potential, ϕ , also obeys
17 Laplace's equation, giving:

18
$$\nabla^2 \phi = 0 \quad (3)$$

19 The disturbance potential satisfies Green's third identity,
20 yielding a Fredholm integral equation of the second kind along
21 the cavitator, cavity, endplate and body. Thus, at any point, x ,
22 on the body-cavity surface, the disturbance potential can be
23 computed from:

$$2\pi\phi(x) = \iint_S \left[\phi(x) \frac{\partial}{\partial n} G(x; x') - \frac{\partial}{\partial n} \phi(x) G(x; x') \right] dS \quad (4)$$

where x' are the points where the sources and dipoles are distributed under the boundary element model;

S is the body-cavity surface; and

$G(x, x')$ is the Green function.

The Green function is further identified as:

$$G(x, x') = \frac{1}{|x - x'|} \quad (5)$$

The dynamic condition on the cavity boundary is derived from Bernoulli's equation. Along the cavity surface, this can be written as:

$$p_\infty + \frac{1}{2}\rho U_\infty^2 = p_c + \frac{1}{2}\rho U_s^2 \quad (6)$$

where p_∞ is the free stream ambient pressure;

ρ is the free field fluid density;

p_c is the pressure inside the cavity; and

U_s is the flow velocity at the cavity surface.

The flow velocity at the cavity surface can be obtained from equation (6) giving:

$$U_s = \sqrt{1 + \sigma} \quad (7)$$

where σ is the cavitation number which is defined as:

$$\sigma = \frac{p_\infty - p_c}{\frac{1}{2}\rho U_\infty^2} \quad (8)$$

The kinetic boundary condition is that no flow crosses the body-

1 cavity boundary,

$$2 \quad \frac{\partial \phi}{\partial n} = -n_x \quad (9)$$

3 where n_x is the axisymmetric body free-stream velocity power.

4 The no net flux condition,

$$5 \quad \oint_S \frac{\partial \phi(x)}{\partial n} dS = 0 \quad (10)$$

6 is also required to make the problem a determinate system..

7 Total drag is calculated by adding the drag coefficients.

8 The pressure drag coefficient, C_p , at \bar{x} is calculated as
9 follows:

$$10 \quad C_p = 1 - U(\bar{x})^2 \quad (11)$$

11 The pressure contribution to the drag coefficient may then be
12 computed as:

$$13 \quad C_{dp} = \frac{4}{\pi} \oint_S C_p n_x dS \quad (12)$$

14 The viscous contribution to the drag coefficient along the wetted
15 portions of the conical and cylindrical body areas is calculated
16 using the International Towing Tank Conference equation given by
17 Newman, Marine Hydrodynamics, MIT Press, Cambridge, Mass. 1980,
18 for the friction coefficient, C_f , at \bar{x} is as follows:

$$19 \quad C_f = \frac{0.075}{(\log_{10}(R(\bar{x}) - 2))^2} \quad (13)$$

20 where $R(\bar{x})$ is the local Reynolds number.

21 The total viscous drag coefficient, C_{dv} , is:

$$C_{dv} = \frac{4}{\pi} \oint_S C_f s_x dS \quad (14)$$

The base drag coefficient, C_{db} , which is the component of pressure drag associated with the base of the body is:

$$C_{db} = \frac{0.029(2b_{base})^3}{\sqrt{C_{dv}}}, \quad (15)$$

where b_{base} is the body radius at the base.

The total drag coefficient is then given by

$$C_d = C_{dp} + C_{dv} + C_{db}. \quad (16)$$

The panels are distributed along the cavitator, cavity, endplate, and cylindrical body section aft of the cavity, according to the partial floor method, known in the art. The partial floor method optimizes the number of panels in accordance with requirements for getting good convergence. Non-uniform panel spacing is used in many locations, in order to reduce the number of panels without reducing the accuracy of the solution.

During iteration, the end plate height is determined by integrating the cavity surface back from its detachment point on the cavitator, and the number and distribution of panels along the endplate changes according to the changes in the endplate height. Smaller panels are required at highly non-linear flow locations, such as the region near the cavitator. Panel distribution in the wetted body area after cavity closure changes to keep the aspect ratio of the neighboring panels

1 between 0.5 and 2.0, in order to ensure good accuracy of the
2 results.

3 In following the method of the current invention, first an
4 initial cavity is defined. An arbitrary initial cavity can be
5 chosen as a cone extending from the cavitator edge to an assumed
6 endplate height of 0.2 or 0.3 is sufficient for most cases. In
7 this discussion, the endplate height is measured as the radial
8 offset from the body surface to the last point of the cavity. By
9 applying equation (4) on all panels along the cavity body
10 surface, S , a system of equations is obtained. This system is
11 solved for the disturbance potentials, ϕ , along the wetted
12 portions of the boundary and on the Riabouchinsky endplate; the
13 normal derivative of the disturbance potential along the cavity
14 boundary; and the cavitation number.

15 The kinetic boundary condition given in equation (9) is
16 applied along cavitator, endplate, and aft body to update the
17 cavity shape. In order to update the cavity, the program
18 calculates how much each panel has to be rotated to satisfy the
19 no flow condition. The program starts with the first panel at
20 the cavitator and shifts the aft most point of the panel in the
21 radial direction which satisfies the calculated rotation. The
22 panel is rotated with the aft most point. The foremost point of
23 the next panel is then shifted to the same radius as the previous
24 aft most point. This process is continued until the panel
25 adjacent to the endplate is updated. The endplate height is

1 adjusted to the aft most point of the aft cavity panel. The
2 iteration continues until the kinetic boundary condition
3 converges to within a tolerance, giving the cavity shape.

4 From the converged disturbance potential along S, the
5 disturbance velocity components can be calculated:

$$u_x = \frac{\partial \phi}{\partial x} \text{ and } u_r = \frac{\partial \phi}{\partial r}. \quad (17)$$

6
7 Referring now to FIG. 2, there is shown a flowchart of the
8 current invention. In the input step 30, geometric and other
9 system parameter data including the estimated cavitation number,
10 the estimated cavity length and the convergence criteria is read.
11 The routine then distributes boundary element panels along the
12 cavitator, cavity, endplate, body extension in the conical
13 section, body extension in the horizontal section, and the aft
14 body. The panels are distributed in order to reduce the number
15 of panels and get an accurate result. In the initialize step 32,
16 the algorithm calculates the unit dipole and unit source
17 functions and initializes matrices for the influence functions
18 with known boundary values wherever applicable. The formulate
19 equations step 34 formulates matrices for each panel using the
20 disturbance potential equation (4) and no net flux condition
21 given in equation (9). The solve equations step 36 solves the
22 matrices created in the formulate equations step 34 in order to
23 obtain the unknown disturbance potential along wetted body
24 sources, normal distributions of disturbance potentials along
25 cavity surfaces, and the cavitation number. The compute forces

1 step 38 computes velocity components such as those in equation
2 (17) and drag coefficients: including pressure drag, equation
3 (12); viscous drag, equation (14); and base drag, equation (15)
4 from the solved equations. In the update cavity step 40, the
5 cavity is updated from the computed forces using the kinetic
6 boundary condition of equation (9). Convergence on cavity shape
7 is checked in the converges decision step 42. If the cavity is
8 not converged, the initialize step 32 is executed to calculate
9 influence functions for the updated cavity and next iteration
10 thus begins. Once the cavity has converged, the compute
11 parameters step 44 computes various output parameters of the
12 converged solution which include pressure drag, viscous drag,
13 base drag, total drag, cavitation number, cavity length, maximum
14 cavity radius, length of cavity to maximum radius location. The
15 output results step 46 then provides the location of the cavity
16 written as coordinates and the cavity's disturbance potential,
17 disturbance potential gradient, and pressure coefficient.

18 The basic algorithm enumerated above provides cavity shape
19 and cavitation number based on an input cavity length. In order
20 to obtain cavity shape and cavity length for an input cavitation
21 number, the embodiment of FIG. 3 adds an additional series of
22 iterations. The user inputs a cavitation number and an assumed
23 cavity length. This embodiment follows the previous embodiment
24 in converging on a new cavitation number, σ , for the assumed
25 cavity length. In step 48, if the new cavitation number is

1 within a tolerance of the given cavitation number, parameters are
2 computed, step 44, and the results are provided, step 46.
3 Otherwise the embodiment proceeds to step 50 wherein the
4 algorithm determines the relationship between the new cavitation
5 number, σ , and the given cavitation number. In step 52, cavity
6 length is increased by a predetermined amount if the calculated
7 cavitation number is lower than the initial cavitation number,
8 and in step 54 the cavity length is decreased by a predetermined
9 amount if the calculated cavitation number is greater than the
10 initial cavitation number. The routine loops back to the
11 initialize step 32 and recalculates the cavitation number for the
12 new cavity length. Operation continues until the calculated
13 cavitation number falls within a tolerance of the initial
14 cavitation number, the cavity length has converged, as tested in
15 step 48.

16 Using this invention, partial cavitation for high-speed
17 underwater bodies can be analyzed. As disclosed, the invention
18 can analyze axisymmetrical bodies using two cavitator shapes, a
19 disk and a cone; however, the invention can easily be modified to
20 analyze other axisymmetric cavitator shapes. As disclosed the
21 inventive method can converge on cavity length or cavitation
22 number. Total drag is calculated by adding the pressure drag,
23 viscous drag and base drag. The invention can also be utilized
24 for studying the effects of body aft radius, body cone angle and
25 body cone angle starting at the cavity closure if the closure is

1 on conical section 22. This method provides new information
2 concerning the physics of cavitation which can be used in the
3 design of cavitating vehicles.

4 In light of the above, it is therefore understood that
5 within the scope of the appended claims, the invention may be
6 practiced otherwise than as specifically described.

What is claimed is:

1. A method for calculating parameters for an axisymmetric partially cavitating body having a cavitator located at the foremost end, said method comprising the steps of:

receiving system parameter data including geometric data describing the axisymmetric body, a convergence tolerance, and an initial cavity shape including an endplate height, an endplate location, and a cavity length;

initially distributing boundary element panels along the initial cavity shape, endplate and the axisymmetric body aft of the endplate;

initializing matrices for each boundary element panel using the disturbance potentials at the boundary element panels and known boundary values;

formulating disturbance potential matrices for each boundary element panel utilizing disturbance potential equations and no net flux boundary conditions;

solving initialized matrices and formulated disturbance potential matrices for each boundary panel to obtain unknown panel sources, unknown dipoles and unknown cavitation numbers;

computing forces and velocities at each panel from the panel sources, dipoles and cavitation numbers to obtain velocity components, pressure drag, viscous drag, and base drag;

updating the cavity by moving each panel in accordance with the calculated forces and velocities and the boundary conditions;

testing for convergence by comparing the movement of each panel against the convergence tolerance;

iterating said steps of initializing matrices, formulating matrices, solving formulated matrices, computing forces and velocities, updating the cavity and testing for convergence when said test for convergence indicates that movement of at least one panel exceeds the convergence tolerance;

computing parameters of interest when said test for convergence indicates that movement of all panels is within the convergence tolerance; and

outputting the location of the cavity and the computed parameters.